Factoring the Elasticity of Demand in Electricity Prices

Daniel S. Kirschen, Goran Strbac, Pariya Cumperayot, and Dilemar de Paiva Mendes

***Abstract*—As electricity markets are liberalized, consumers become exposed to more volatile electricity prices and may decide to modify the profile of their demand to reduce their electricity costs. This paper analyzes the effect that the market structure can have on the elasticity of the demand for electricity. It then describes how the consumers' behavior can be modeled using a matrix of self- and cross-elasticities. It is shown how these elasticities can be taken into consideration when scheduling generation and setting the price of electricity in a pool based electricity market. These concepts are illustrated using a 26-generator system.**

***Index Terms*—power system economics, elasticity, generation scheduling, pricing.**

I. INTRODUCTION

FLATTENING the load curve has long been recognized by utilities as an effective way of cutting the cost of producing electricity. To encourage their consumers to modify their demand pattern in a beneficial way, they have adopted a variety of demand-side management measures such as two-part tariffs and interruptible load contracts. While these special tariffs are mutually beneficial, it is not always clear that the benefit for the consumer is proportionate to the value they provide to the utility.

The liberalization of the electricity markets has led in many parts of the world to the replacement of tariffs by hourly or half-hourly prices. Economists argue that these prices are a powerful way to encourage consumers to behave in an economically optimal way. One must make a distinction between the long and short-term effects of such prices. In the long term, the average price will affect the overall level of consumption. Wide differences in prices between day and night or between weekends and weekdays may also encourage customers to install thermal or material storage that will help them avoid consuming electricity during the hours of peak prices. Computing the value of such investments and the long-term effects that they will have on prices are interesting and complex problems that this paper will not address.

In the short term, some customers have the ability to reduce or reschedule their demand in response to the electricity prices. For example, if prices are high, some industrial consumers may forego production if it is not profitable at that price level. Consumers who have the ability to store energy or some intermediate product may reorganize their production. Schweppe and his co-workers [1] formalized these ideas and developed the concept of spot pricing of electricity. They envisaged a system where customers would adjust their demand up or down depending on the spot price. The spot prices would be updated in real time to take into account these load adjustments. They demonstrated that this approach maximizes the global welfare. Furthermore, volatile prices increase the value of storage facilities that help consumers avoid periods of high prices. This encourages consumers to invest more in such facilities and contribute further to the global welfare.

Schweppe's work is widely regarded as having provided the theoretical foundation for the current liberalization of the electricity trade. However, none of the competitive markets that have been implemented so far has given consumers the opportunity to reduce their demand in response to spot prices and see this response affect the prices. Neglecting the consumers' reactions when setting electricity prices has lead to some absurd and patently unfair situations. For example, on several occasions [2], a combination of high forecasted demand and generating unit unavailability has led the scheduling program used by the Electricity Pool of England and Wales to schedule some peaking units which had submitted very high bids. Consequently, the price of electricity for the few hours during which these units were scheduled was extremely high even though the amount of power to be produced by these units was very small. Since these prices were published more than 13 hours before they took effect, some large industrial consumers decided that production at those prices was not profitable. They rescheduled their production or simply curtailed their demand by a sufficient amount that, had this reaction been taken into consideration, the prices would have been much closer to normal values.

Letting the consumers demand affect the price of electricity in real-time, as proposed by Schweppe and his co-workers, is the theoretical answer to this problem. Unfortunately, this approach does not appear practical at this point. David and others [3]–[5] did a considerable amount of theoretical work in the late 80's and early 90's on the elasticity of customer demand. However, that work was done before any significant experience was gained on the operation of competitive electricity markets and the setting of short-term prices. More recently, Rajaraman et al. [6] have shown how elasticity should be taken into account in the calculation of security prices. The purpose of this paper is therefore to explore, in the light of the experience gained since then, how the short-term elasticity of the demand for electricity could be taken into consideration when scheduling generation and setting prices.

Section II of this paper reviews the method used for setting prices in a competitive electricity market and discusses how the design of this market affects the consumers' ability to influence

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these prices. Section III describes how the elasticity of the demand for electricity can be modeled. Section IV shows how this model can be incorporated in a scheduling and pricing program. Finally Section V illustrates this integration with test results.

II. ELECTRICITY MARKETS AND PRICING

Competition in bulk electricity supply always revolves around an organized market or pool whose primary purpose is to discover the price of electricity for the upcoming time intervals. In essence the balance of supply and demand sets the price as follows:

- Generators offer bids for a certain amount of power at a certain price
- These bids are ranked in terms of their price
- Bids are taken in this order until the demand is satisfied
- The last accepted bid sets the market price.

The precise shape of this pool is determined by a number of key design decisions that have an effect not only on the price of electricity but also on the reaction of the consumers to these prices [2], [3]. The following paragraphs discuss these design decisions.

Optional versus Compulsory Pool: Is it compulsory to trade all electricity through the pool or are physical bilateral contracts permitted? If participation in the pool is compulsory, consumers are fully exposed to the pool price. On the other hand, if participation is optional, consumers may enter into contracts with producers that shield them from the vagaries of the pool price. (Note that even in a compulsory pool, consumers are free to enter into financial contracts with generators or third parties. These contracts for difference insulate consumers from the price volatility just as effectively as physical contracts.)

One- or Two-Sided Market: Are there bids for both supply or demand or is demand taken as a constant determined on the basis of a load forecasting program? Letting consumers enter bids for their demand either directly or through aggregators gives them the opportunity to indicate the value they put on their load. There is, however, very little experience with demand side bids on electricity markets. It is therefore not clear how these bids affect the actual behavior of the consumers.

Firmness of Bids and Offers: Are the bidders required to meet the physical commitment to generate or purchase electricity? Equivalently, are they exposed to financial risks if they do not deliver or take delivery of the quantity they bid? Consumers that are exposed to a large volatility in prices will definitely pay more attention to their demand profile than those who buy on a flat tariff.

Simple or Complex Bids: Do the bids involve only a single quantity and a price or are they designed to reflect the various elements of the cost of running a generating unit? In other words, does the generator assume the risk associated with the start-up and no-load costs of its unit or is this risk passed on to the pool? In theory, centrally scheduling generation on the basis of complex bids should lower the overall cost of generation, reduce the risk for the generators and hence result in lower electricity prices. Experience with the Electricity Pool of England and Wales (EPEW) has shown that this approach has two major flaws. First, rather than minimizing the price of electricity for a given level of demand as an efficient market would, this simulated market minimizes the production cost. Second, this cost minimization can result occasionally and unexpectedly in large price increases over a short period [3]. While a market based on simple bids could also lead to very high prices, it would seem that its behavior is likely to be more predictable.

Market Timing: Is the price determined before (ex ante), after (ex post) or at the same time as the delivery of electricity takes place? If the price is determined ex ante, how much time elapses between the determination of the price and the delivery? From an economist's perspective, fixing prices ex post is deeply unsatisfactory. It also precludes any reaction from the consumers, as they won't know the price until it is too late to adjust consumption. Adjusting prices in real time or very close to real time could lead to a true interaction between supply and demand in the setting of the price. Unfortunately, the scope for real time demand adjustments appears very limited at this point. Setting the price in advance gives consumers time to adjust their activities and their demand profile. As discussed below, a mechanism must then be found to factor in these demand adjustments in the prices.

Capacity Payments: In some markets, generators are paid for making generation capacity available, irrespective of the amount of electrical energy that this generation capacity actually produces. Since these capacity payments are intended to encourage generators to keep marginally profitable generators available, they usually increase nonlinearly as the difference between the load and the available capacity decreases. These payments therefore tend to increase sharply during periods of very high loads.

Geographically-differentiated Pricing: Is a uniform price used for the entire system or is the effect of network constraints translated into geographical differences in prices? Network constraints are likely to increase the volatility of electricity prices in some parts of the network. Consumers in these parts will therefore pay closer attention to their consumption profile.

Price Capping: In the EPEW, the price of electricity is capped at an officially determined Value of Lost Load (VOLL) that is supposed to represent the average value of electricity for the consumers. The justification for this price cap is that economically rational customers should stop consuming if the price were to exceed that value. This is a very crude way of taking into account the customers' perception of electricity prices. VOLL is indeed averaged over a wide variety of customers and reflects the cost of unexpected interruptions rather than the cost of reorganizing or reducing production [9]. This price cap has never been activated in the EPEW.

As the above discussion illustrates, a number of factors influence the customers' response to electricity prices. Furthermore, the structure of the market determines how the natural elasticity of electricity demand is or should be taken into consideration. In a highly flexible market where each customer can negotiate its own prices and generators schedule their own production, elasticity would be factored in automatically and would not have to be addressed centrally. Such decentralized markets do not exist yet. In pool based markets, such as the EPEW, elasticity must be taken into account centrally as it affects both the price of electricity and the scheduling of generation. The rest of this

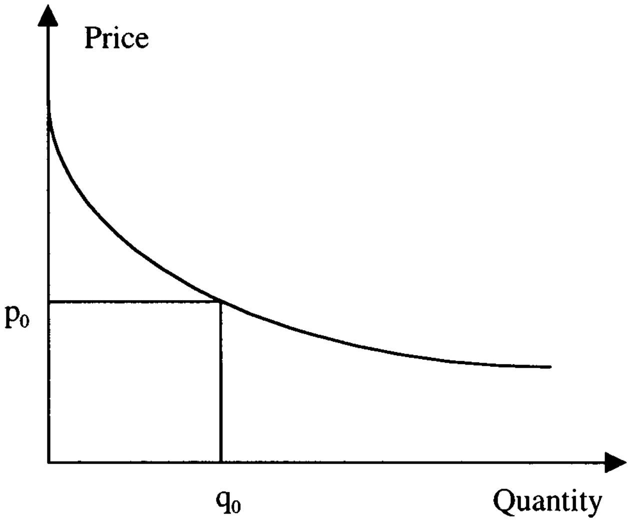

Fig. 1. Typical demand curve.

paper discusses elasticity in the context of a market similar to the EPEW. The EPEW can be defined as a "compulsory, one-sided, nonfirm market in which complex bids are used to set market prices on a marginal, ex-ante basis with the cost of imbalances averaged and an additional capacity payment levied" [8]. In this market, half-hourly prices are announced 13 hours in advance of the beginning of the next 24-hour scheduling period.

III. PRICE ELASTICITY OF ELECTRICAL DEMAND

The demand for most commodities decreases as the price of the commodity increases as illustrated by the demand curve sketched on Fig. 1. As this curve is difficult, if not impossible, to quantify, economists often linearize this curve around a given point. They then define the price elasticity of demand as the relative slope of this demand curve:

$$\varepsilon = \frac{\frac{\Delta q}{q_0}}{\frac{\Delta p}{p_0}} \tag{1}$$

This elasticity coefficient indicates the relative change in demand for a commodity that would result from a change in the price of this commodity.

In the remainder of this paper, it will be assumed that all prices and quantities have been normalized with respect to a given equilibrium point (q_0, p_0). The elasticity can then be expressed as:

$$\varepsilon = \frac{\Delta q}{\Delta p} \tag{2}$$

In some cases, a change in the price of one commodity will affect the demand for another commodity. For example, an increase in the price of coffee will reduce the demand for coffee but may increase the demand for tea. A negative "self-elasticity"can be used to represent the first effect and a positive "cross-elasticity" the second.

$$\begin{cases} \Delta q^a = \varepsilon_{aa}\Delta p^a; & \varepsilon_{aa} \le 0 \\ \Delta q^a = \varepsilon_{ab}\Delta p^b; & \varepsilon_{ab} \ge 0 \end{cases} \tag{3}$$

If the reciprocal effects between price and quantities of these two commodities are of interest, an elasticity matrix can be defined:

$$\begin{pmatrix} \Delta q^a \\ \Delta q^b \end{pmatrix} = \begin{pmatrix} \varepsilon_{aa} & \varepsilon_{ab} \\ \varepsilon_{ba} & \varepsilon_{bb} \end{pmatrix} \begin{pmatrix} \Delta p^a \\ \Delta p^b \end{pmatrix} \tag{4}$$

As argued above, the customers' reaction to changes in the price of electricity depends on the time frame considered. This paper focuses on the short-term response where short-term will be defined as the time that elapses between the publication of the prices for the next 24-hour interval and the actual demand periods. In the EPEW, the consumers have between 13 and 37 hours to reschedule or curtail their production. With respect to the demand for electricity, a self-elasticity coefficient relates the demand during a half-hour period to the price during that half-hour. A rescheduling of production implies that the consumer reduces its electricity demand during some half-hours and increases it during other. Cross-elasticity coefficients relate the demand in one half-hour to the price during other half-hours. The change in demand at half-hour i caused by a deviation of the published prices from the prices expected by the consumers is therefore given by:

$$\Delta q_i = \sum_{j=1}^{48} \varepsilon_{ij}\Delta p_j \tag{5}$$

If it is assumed that the reorganization of the production does not extend beyond the 24-hour scheduling period, these self-and cross-elasticity coefficients can be arranged in a 48 by 48 matrix E:

$$\Delta Q = E\ \ \Delta P \tag{6}$$

The diagonal elements of this matrix represent the self-elasticities and the off-diagonal elements correspond to the cross-elasticities. Column j of this matrix indicates how a change in price during the single period j affects the demand during all the periods. If the only nonzero elements in this column are above the diagonal, the consumers react to a high price by bringing forward their consumption. If they are below the diagonal, they postpone their consumption until after the high price period. If consumers have the ability to reschedule their production over a long period, the nonzero elements will be spread widely over the column. On the other hand, if their flexibility is limited, the nonzero elements will be clustered around the diagonal. Some customers may also decide that, if they have to reschedule their electricity consumption, they might as well take advantage of the hours of lowest prices, which typically are in the early hours of the morning. Fig. 2 illustrates the structure of the elasticity matrices corresponding to these various types of consumer reactions.

The consumers' ability to react to unusual electricity prices varies with the time of day. One column of the elasticity matrix

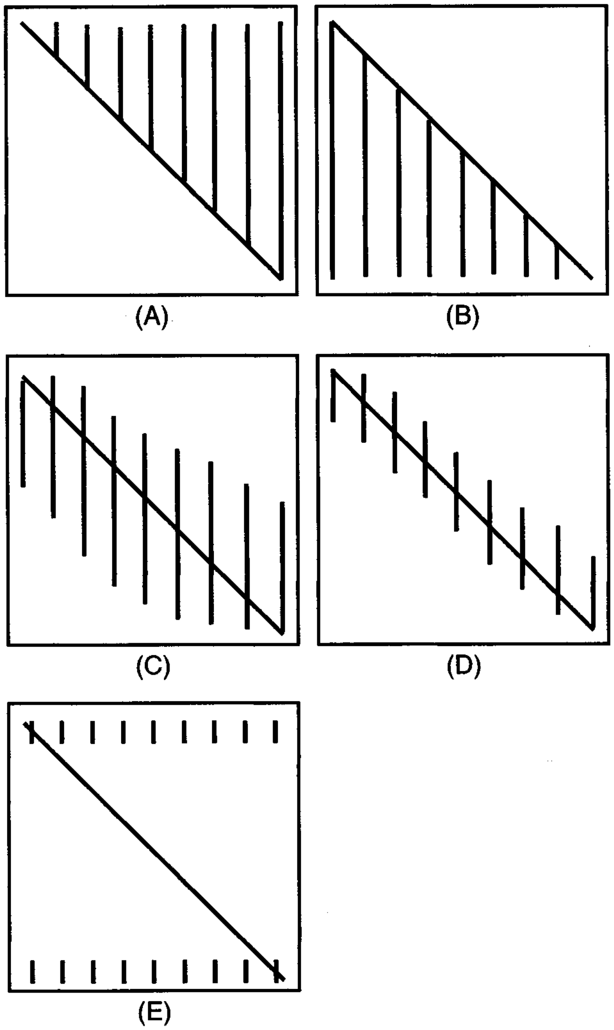

Fig. 2. Structure of the matrix of elasticities for various types of customer reactions. A: Anticipating consumer. B: Postponing consumer. C: Flexible consumer. D: Inflexible consumer. E: Optimizing consumer.

can therefore not be deduced from another through a simple translation along the diagonal. It should also be noted that the elasticity matrix relates changes in demand to changes in prices within a single scheduling period. Changes in demand due to unusual prices in a previous period must be carried over separately in the load forecast.

If an unusually high price induces a consumer to reorganize its production without a reduction in energy demand over the 24-hour scheduling period, the following relation holds between the elements of each column of the elasticity matrix:

$$\sum_{j=1}^{48} \varepsilon_{ij} = 0 \quad \forall i \tag{7}$$

An elasticity matrix that obeys (7) will be called lossless. On the other hand, if the consumer reduces its demand, this relation becomes:

$$\sum_{j=1}^{48} \varepsilon_{ij} < 0 \quad \forall i \tag{8}$$

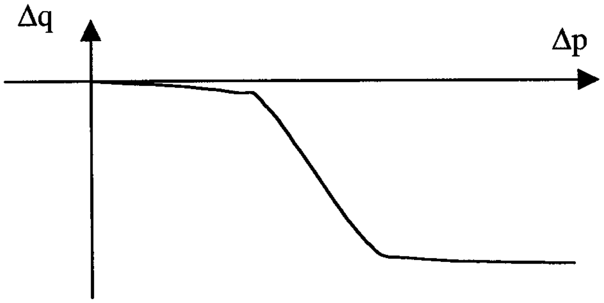

Fig. 3. Non-linear elasticity function.

In practice, the set of all consumers consists of a mixture of all the types described above. Therefore, the structure of the elasticity matrix and the value of its elements have to be determined through the analysis of the consumers' response to actual deviations of prices from their expected values.

Cutting back on electricity consumption involves at least one of the following activities: reorganizing production, adjusting controls, using energy or intermediate product storage systems, calling upon backup generation or substitute energy sources, cycling equipments. Since all these options are relatively cumbersome, most consumers are unlikely to react to an increase in price until this increase becomes significant.

There is also a level beyond which load reductions become very difficult if not impossible to implement. Furthermore, customers are much less likely to increase or reorganize their production to increase their consumption of electricity in the case of a short-term price drop than they are to react to a price increase. Non-linear elasticity functions such as the one shown on Fig. 3 have therefore also been implemented.

IV. INTEGRATION WITH A SCHEDULING PROGRAM

Fig. 4 illustrates how the elasticity of the demand for electricity can be taken into consideration when the price of electricity is set by a centralized, compulsory pool which schedules generation on a half-hourly basis for a 24 hour period. As the figure suggests, the computation of the effect of the elasticity must be iterated with the scheduling algorithm and the price computation. This implementation differs from the one described in [4] in two important aspects:

- Generation is scheduled using a unit commitment program instead of an optimal power flow. Experience with the EEPEW has indeed shown that start-up costs can create very significant price spikes. The unit commitment program has been implemented using a Lagrangian Relaxation algorithm [10].
- The price computation is carried out according to the rules of the Electricity Pool of England and Wales [11].

V. TESTING

The proposed method has been tested using a 26-generator system [12], [13] over 24 1-hour periods. First, the vector of expected prices was determined by running the unit commitment

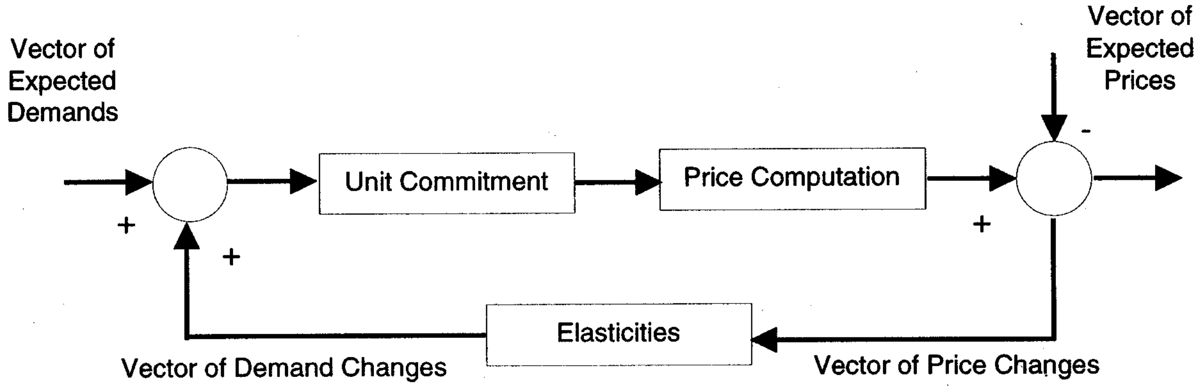

Fig. 4. Integration of the elasticities with the price computation.

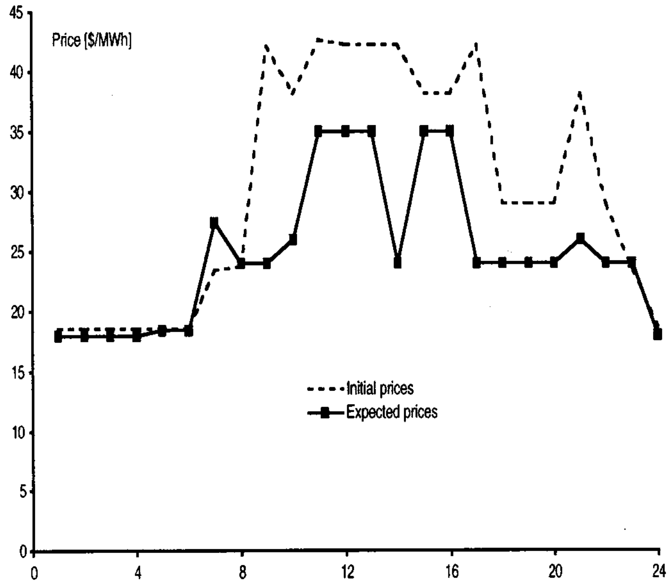

Fig. 5. Expected prices and initial prices.

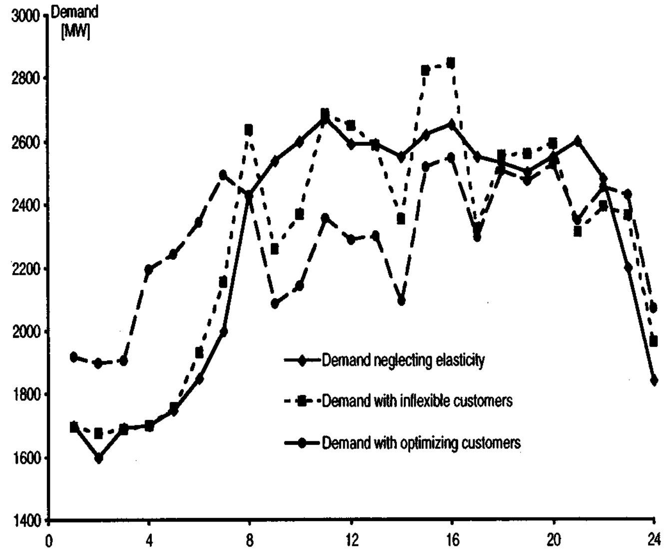

Fig. 6. Original demand and demand as modified by elasticities.

and pricing programs with all the units available. In order to simulate a realistic increase in prices, these programs were then run assuming that one large, base generation unit was unavailable.

Fig. 5 shows the expected prices and the initial prices, i.e. the prices as they would be if the elasticity was not considered and a large unit was not available. Fig. 6 shows the original demand as well as the demand as it would be if the elasticity were modeled. Two cases are considered: in the first the consumers are assumed to be inflexible and the demand is shifted to the three hours immediately preceding and following each period. The self-elasticity coefficient was set at -0.2 while the cross-elasticity coefficients were all given a value of 0.033 to ensure a lossless situation. In the other, it is assumed that the consumers "optimize" the shift in their demand by moving it to the periods of normally low price (00:00 am to 07:00 am and 04:00 pm to 12:00 pm). The self-elasticity coefficient was again given a value of -0.2 while the cross-elasticity coefficients were given values of 0.01 (for the periods from 00:00 am to 03:00 am and 04:00 pm to 12:00 pm) and 0.025 (for the period from 04:00 am to 07:00 am). Fig. 7 shows the effect that this redistribution of the demand has on the prices. As one would expect, the demand optimization results in considerably lower prices. In this example, where the elasticities are most likely exaggerated, the total savings to consumers is about 12% while the reduction in cost to generators is about 2%. It should be noted that the rules of the EPEW are such that generators are always paid at least the amount they bid and will therefore never loose money if their bid reflects their operational cost. If the consumers are inflexible (i.e. if they can only shift their demand by a few hours), the price swings up and down quite significantly. This can be explained by the fact that they reschedule their demand for periods where the load and the prices are already fairly high. In this case, the savings to consumers are considerably smaller than when the demand is shifted to periods of low prices.

VI. CONCLUSIONS

This paper has discussed how the elasticity of the demand for electricity could be taken into consideration when setting the price of electricity in a centralized competitive market. It has also been shown how different types of consumer reactions to

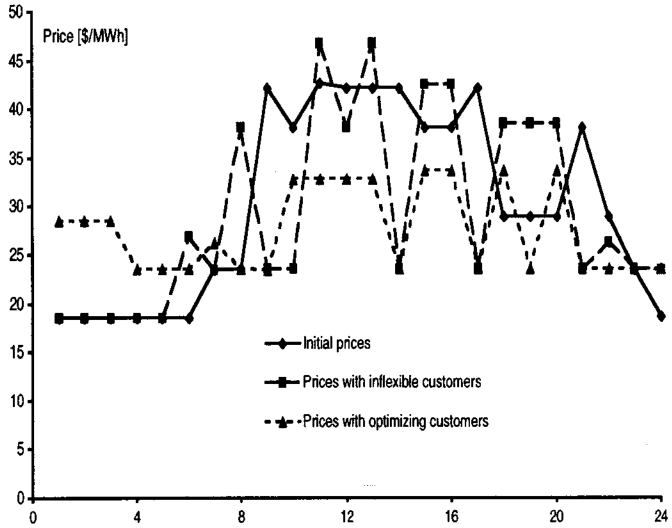

Fig. 7. Initial prices and prices as modified by elasticities.

volatile electricity prices can be modeled using the concept of cross-elasticity.

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